## Bentley and Copdock Calculation Policy

At Bentley and Copdock Schools we wish to teach calculation with understanding, and not just as a process that is to be remembered. This Calculation Policy clarifies progression in calculation with examples that are 'mathematically transparent', in other words the way the calculation works is clear and supports both the development of mathematical concepts and closely links it to the mental strategies that are taught alongside the written methods.

## The Aims of the curriculum:

The national curriculum for mathematics aims to ensure that all pupils:
$>$ become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
$>$ reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
$>$ can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.
We seek to reflect these aims in all mathematics lessons and in other areas of our curriculum where maths skills are being applied.

## Recording

Recording is developed in a range of ways, including the following. Although initially they will be developed in this order, once a way of recording, such as 'by showing real objects', is in place, that will continue to be used throughout the Primary years. In EYFS most recording will be by showing real objects, whereas in Y6 real objects may be used to show an understanding of calculation with decimals.

Development of recording:
$>$ by showing and using real objects
$>$ by photographing or drawing the calculation activity
> counting on a number line
$>$ a practical calculation activity on a number line
$>$ a number bond on a number line
> a mental calculation on a number line
$>$ a practical activity as a number sentence
> a number bond as a number sentence
$>$ a mental calculation as a number sentence
$>$ a written calculation - informal jottings
> a written calculation - expanded methods
> a written calculation - compact methods

We will always seek to set the calculations in real life contexts and will provide a range of representations, models and physical resources to support the conceptual development of the calculation. These representations, models and physical resources are common throughout the school and build successively over the year groups. Children are encouraged to work through concrete to pictorial to abstract representations. Access to them will be encouraged in all classes and the children will play an active role in deciding when, which one or if to use them. The written calculations will be written alongside the images and representations using "What's the Same, What's Different?" strategies to make connections between the written calculation and the resources or representations used.

Teachers will provide whole class, group and individual support when directly teaching calculation strategies. These will be embedded with regular hands-on activities to practice the skills in context. They will also apply these skills in a wide range of problem solving activities; starting with problems that strongly

## Progression in key mental facts

## By end of Reception

- Pairs of numbers (called number bonds) that make each number to 5 and some numbers to 10 (e.g. $0+5,1+4,2+33,3+2,4+1,5+0$ are all number bonds that make 5).
- Know double facts to 10 (e.g. double $4=8$ ).
- Find 1 more or 1 less than any number to 10.


## By end of Year 1

- Addition facts of pairs of numbers (number bonds) that make each number to 20 and the associated subtraction facts (e.g. $8+7=15,7+8=15,15-7=8,15-8=7$ ).
- Know double facts to 20 (e.g. double 8=16).
- Count in multiples of 2 to 20,5 to 50 and 10 to 100 (e.g. multiples of 2 are 2, 4, $6,8,10,12,14,16,18,20)$.
- Find 1 more or less than any number to 100 .


## By end of Year 2

- Recall addition and subtraction facts to 20 and related facts to 100 (e.g. $6+4=10$ so $60+40=100,10-3=7$ so $100-30=70$ ).
- Know double facts to 20.
- Count in multiples of 2, 3, 5 and 10 forwards to 100 and backwards.
- Recall $x$ and $\div$ facts for $2 x, 3 x, 5 x$ and $10 x$ tables (e.g. $7 x 5=35$ and $35 \div 5=7$ ).
- Find 10 more or 10 less than any number to 100 .


## By end of Year 3

- Count forwards and backwards in multiples of 2, 3, 4, 5, 8, 10, 50 and 100.
- Recall and use $x$ and $\div$ facts for $2 x, 3 x, 4 x, 5 x, 8 x$ and $10 x$ tables (e.g. $4 x 8=32$ and $32 \div 8=4$ ).
- Use doubling to learn the $2 x$ (double), $4 x$ (double double) and $8 x$ (double double double) tables.
- Find 10 or 100 more or less than any number.
- Multiply a 2 digit number by 10 (e.g. $37 \times 10=370$ ).
- Double and halve multiples of 10 to 100 (e.g. double $30=60$, halve of $80=40$.
- Add and subtract multiples of 10 to 100 (e.g. 30+40=70, 90-60=30).


## By end of Year 4

- Count forwards and backwards in multiples of $2,3,4,5,6,7,8,9,10,25,50$, 100 and 1000.
- Recall and use $x$ and $\div$ facts for all tables up to $12 x$ table.
- Find 1000 more or less than any number.
- Add and subtract multiples of 10 or 100 (e.g. 700-400=300)
- Multiply a 2 digit or 3 digit number by 10 or 100 (e.g. $345 \times 100=34500$ ).
- Double and halve multiples of 10 to 100 (double 70=140, halve of 500=250).
- Halve any even number to 100 e.g. halve of 22=11)


## By end of Year 5

- Count forwards and backwards in multiples of 100, 1000, 10000, 100000 and 1000000.
- Add and subtract multiples of 100 and 1000 (e.g. 500+700=1200, 6000$2000=4000$.
- Double and halve multiples of 10 to 100 (e.g. double $60=120$, halve of $90=45$ ).
- Quadruple (x4) all numbers to 10 (e.g. quadruple $6=24$ ).
- Multiply 2 digit numbers by 10 (e.g. 37x10=370).
- Halve any number to 100 (e.g. half of $37=18.5$ ).
- Multiply and divide any number by 10 and 100 (e.g. $45 \div 10=4.5,3.4 \times 10=34$ ).
- Square numbers up to $12^{2}$ (e.g. $1,4,9,16,25,36, \ldots$ )
- Cubes of $2,3,4,5$ (e.g. $2^{3}=8,3^{3}=27,4^{3}=64$ and $5^{3}=125$ ).


## By end of Year 6

- Add and subtract multiples of 10,100 and 1000 (e.g. $500+700=1200,6000-$ $2000=4000$.
- Double and halve multiples of 10 to 100 (e.g. double $60=120$, halve of $90=45$ ).
- Quadruple (x4) all numbers to 10 (e.g. quadruple $6=24$ ).
- Multiply 2 digit numbers by 10 (e.g. 37x10=370).
- Halve any number to 100 (e.g. half of $37=18.5$ ).
- Multiply and divide any number by 10 and 100 (e.g. $45 \div 10=4.5,3.4 \times 10=34$ ).
- Square numbers up to $12^{2}$ (e.g. $1,4,9,16,25,36, \ldots$ )
- Cubes of $2,3,4,5$ (e.g. $2^{3}=8,3^{3}=27,4^{3}=64$ and $5^{3}=125$ ).


## Progression in calculation

## Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same value as', partition, exchange, "carry", number sentence.

## Concrete:

Children begin calculation purely with practical activities. Over time they learn to record these activities in a way that makes sense to them. This will be by showing or taking photographs of the equipment they have used, leading to drawings of what they did.
For instance, with the practical activity - I have 3 sweets, then I get one more. The child draws the sweets. They may draw 3 sweets and then another. They may just draw 4 to start with.
This means that any recording of the format $3+1=4$ is very unhelpful and is not based on their experience but on an abstract recording method.

The children may choose to use Numicon to add, instead of real objects.


The children may choose to make the numbers out of straws bundled into tens for easy counting. These straws are used as a precursor to number rods. The children will make the two numbers out of straws and then push them together to combine them. In the example below, they will need to put an elastic band around 10 of the loose straws to make another bundle of 10 , to show the answer is 22 .
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As well as using objects, pupils will begin to use number tracks and then number lines both as practical equipment that makes the calculation transparent and as ways to record what they did.
For calculations it is useful to have 'lollipop' number tracks, where counters can be placed in the circles without covering over the numerals.
3 plus 1, for instance:


At first children will record their counting on number lines, later moving to recording of calculation on a number line. Pupils will use numbered number lines to record jumps, for example for $3+2$, before recording on blank number lines.


By the end of Y1 children should be confident using number lines to 'play' with numbers. They learn that maths is about playing with numbers and trying things out rather than just finding the right answer.

## Recording number sentences

Before pupils move to recording $3+1$ they will need lots of experience of practical addition, and an ability to respond to mathematical vocabulary practically. For instance, if you ask a child to show you 5 and 2 more, or 5 plus 2 , or 5 add 2 , they can use the teddies, counters or number tracks to show you. They will also be developing their use of mathematical vocabulary to explain what they have done.
From this it will be possible to develop an understanding of the + sign, which will enable pupils to begin to record in the form 5+2.

Pupils then need to understand the concept of equality before using the = sign. This means they can see an example such as $7=6+1$, or $5=5$, as well as the more common arrangement $3+1=4$, and know that it makes sense, because both sides are equivalent or equal.

Pupils will still work practically with equipment and real objects, but now can record their explanation of what they have done as a conventional number sentence:

$$
3+14=17 \quad 17=14+3 \quad 17-3=14 \quad 3+14=14+3 \quad \text { and so on. }
$$

However, pupils will still record with objects, drawings and number lines on a frequent basis, and whenever they are learning new concepts or starting to use a wider range of numbers they will need to return to using these easily understood and explained methods of recording.

## Pictorial:

## Mental methods

Pupils need to develop their use of jottings to support mental calculation. These jottings may be as drawings, number lines or number sentences.

$3+4=7$ using part part whole model

$4+1+1$ as a bar model

$6+5=11$ using a tens frame

$36+25=61$

$243+368=611$
Children may draw base 10 equipment or draw the place value counters they would use to solve an addition.

Once children have an understanding of place value in 2-digit numbers, in other words they are convinced that 23 is 20 and 3 , or 59 is 50 and 9 , they can begin to use partitioning in their mental and recorded calculations.

## Abstract:

Children can begin to record their calculations using part part whole models and abstract number lines.

$40+1=41$


What is 2 more than 4 ? What is the total of 4 and 2 ?

Children use missing number symbols to develop an understanding of equality e.g.
$6+\square=11$
$6+5=5+\square$
$6+5=\square+4$

This then leads to partitioning.

## Partitioning

Partitioning may be recorded in a number of ways, such as:

```
36+45=30+40+6 +5
    = 70 + 11
    =81
```

or

```
36+45=36+40+5
    = 81
```

```
536 + 245=500 + 200 + 30 + 40 + 6 + 5
    =700 + 70 + 11
    =781
```

```
24.5+87.8=20+80+4+7+0.5+0.8
    =100 + 11 + 1.3
    =112.3
```

The important thing to consider when children are recording partitioning is that they record how they thought about the numbers, and don't all try to do it the same way. This is not about finding lots of ways to record, but of recording what makes sense to a child. Partitioning is also an appropriate strategy for larger numbers, eventually including decimals.

## Partitioning using number lines

## Key understanding - A number line is a tool, not a rule.

Children partition numbers to count on, mainly in multiples of 100 , 10 or 1 , on a number line. Number lines will be used for calculations right through Key Stage 2. What matters, however, is that children make their own choices of which numbers to use and that they use their understanding of number and place value to find a way that works for them. This may continue into 3-digit numbers for some children.


Children need to develop understanding of calculation in a range of contexts, for instance measures, including money and time. Time is particularly difficult, and at first children will use number lines to record counting in steps of hours or minutes. Counting across boundaries is particularly important. Number lines are also used for calculating with negative and positive numbers.

When children are solving problems in different contexts, they may use a bar method of representing the calculation so that they can see what they need to do in order to solve the problem.


The bar helps the child to see what they need to do but doesn't work the answer out for them. Now they know what to do, they can use mental methods, jottings or other representations to find the answer

## Expanded vertical method

In Year 4 pupils may begin to record addition calculations vertically (or in Year 3, if able and ready for it), at first recording calculations both as the partitioning they have been using and as an expanded vertical calculation, adding numbers in columns, beginning with the hundreds, then tens and then adding the ones. The vocabulary used will always be whole number place value vocabulary, so 254 would be 200,50 and 4, never 2 hundreds, 5 tens and 4 ones. We always use 'ones', as the term 'units' is only used for units of measurement and not for place value. Children discuss what is the same and what is different about each of these ways of recording. They realise that it doesn't matter in what order you add the totals for the ones, tens or hundreds. The final total is always the same.


This will be supported using number rods and then moving onto Place Value Counters. Written calculations will develop alongside the concrete or pictorial representations for a significant amount of time.


Once pupils are very confident with this method of recording they may extend it to numbers with more digits, providing their understanding of place value is sufficient to support this. There is no need to head each column with $\mathrm{H}, \mathrm{T}$ or O , as writing this does not help pupils who do not understand place value, and is unnecessary for those who do.


It is possible to record the vertical method more quickly by making a note when the addition of two or more numbers goes above 1,10 or 100 and so on, rather than writing it all out. This compact method would not normally be used before Year 5, and even then there is no hurry to move to this. It is best left until children have used partitioning with decimals and are very confident with this.
When children use the 'shortcut' or compact method they need to know that it works in a similar way to partitioning, but that you add the ones first. If children really understand the expended vertical method and have used if for a year or more it should be possible to teach the compact method in one or two lessons.

Column method


Key understanding - It's best to talk about 'making one' from adding 0.2 and 0.9, and putting this with the other ones, and 'making 100' from adding 50+50 and putting this with the other hundreds, rather than 'carrying' one.

## 348 <br> $+235$



| 2 | 7 | 5 | 3 | . | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 6 | 5 | 4 | . | 9 |
| 6 | 4 | 0 | 8 | . | 1 |
| 1 | 1 |  | 1 |  |  |

## Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease, partition, exchange, "swap shop", number sentence.

## Physical:

As with addition, subtraction is initially recorded as drawing the result of a practical activity using equipment to physically take away, or by using Numicon, by laying the smaller number on top to find the difference between them.


The children will then move on to record this using numbers, on number tracks or lines or as number sentences. Initially number tracks or lines will be used to subtract small numbers such as $5-2$ by counting back.


Children can use tens frames to make 10 by subtracting 4 and then subtracting the 1 from 10 to solve 14 - 5 $=9$.



## Key understanding - Pupils need to realise that partitioning is not appropriate for subtraction.

In the example 34-17 it is possible to start with $30-10$, but 4-7 is less useful! The children can use the bundles of straws or number rods to make the first number and then take the second number away. With the straws, the elastic band can be taken off a bundle when there are not enough single straws to take away. This will help the children understand 'exchanging' when they get to the compact written method. We call this going to the "swap shop".


The children move from bundles of straws to physically using base 10 rods and then place value counters.

## Pictorial:

Children begin by drawing the physical equipment they have used before and crossing out the ones they are taking away. They draw pictures of tens frames or base 10 or place value counters, making sure they show where they have exchanged a ten for ten ones.


## Abstract: <br> Key understanding - Putting the zero on a number line for subtraction and crossing out what has been subtracted makes the subtraction obvious.

By the end of Year 2, children use the FROG method for subtraction where they find the difference by counting on. For example, for 82-49, children start at 49 and jump to the next multiple of 10 (50) then jump in tens to 80 , then jump to 82 , counting the total of the hops as they go.


You'll notice that there is a zero placed on the number line. This helps to stop children writing the 73 on the left hand side of the number line, but more importantly enables you to cross out and 'take away' the 26. It makes it easier to understand that this is a subtraction, and you are counting on to find out how many are left. So this use of number lines builds on the understanding of subtraction as difference or as complementary addition.


The jumps on this first number line are in tens and ones. This is a good starting point as it builds on the daily counting that children will be doing, including counting on in tens and ones from any number. It also means that calculating how many you have jumped altogether is easy. Of course children may do different jumps. When they are confident with this stage, pupils can reduce the number of steps.


Subtraction of decimals is just as simple using the number line.
32.4-13.8

$32.4-13.8=18.6$
Don't forget that children will still encounter calculations where it's equally sensible to count back.
3004-96


We continue to use number lines for subtraction calculations in a range of contexts, such as time, money, mass, length and capacity. Often when we have a word problem we put the information into a bar to make sure we understand what we need to do. This doesn't work out the answer for us but helps us see what we need to do. The bar model helps us to see the difference between 2 numbers, as in the example below:


## Vertical calculation for subtraction.

Some children will always achieve success using a number line but in Yr 4 a column method will be introduced. This will begin by being modelled firstly with number rods or bundles of straws, moving onto Place Value Counters.


The written calculation will be alongside the practical representations. The children will need extensive use to begin to feel secure with this method.

## Subtraction with Partitioning.

| 200 | $100+40$ | $10+4$ |
| ---: | ---: | ---: |
| 300 | 50 | -4 |
| - | 100 | 80 |
|  | 8 |  |

Step 14 subtract 8 is possible, but the answer would be negative. Partition the 50 into 40 and 10. Now you can put the 10 with the 4 , so you have enough to subtract 8 without giving a negative answer.
Step 2 You'll notice that it's still $10+4$. There is no need to change this to 14 . If you have both 10 and 4 you can subtract 8 and this leaves 6.
Step 3 The next step is to subtract 80 from 40. Again this would give a negative answer, so partition the 300 into 200 and 100 and put the 100 with the 40.
Check that $200+100$ and 40 and $10+4$ still make the 354 you started with.
Step 4 Now you can subtract 80 from 100+40, which gives 60, and then subtract 100 from 200, leaving 100.

It's possible to move from this, once children have extensive experience working in this way, to a compact decomposition method, though this may not be necessary and may not be an improvement. Place Value Counters would be used alongside the written calculation to support understanding.


## Multiplication

## Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups, equal hops, array, multiple of, number sentence.

## Concrete:

Children's first recording in multiplication will be by placing objects in groups and then in arrays, counting in steps on number lines from zero and using partitioning with base 10, Numicon or place value counters.


Concepts of multiplication develop using doubling and counting in steps, and are extended using the array. Objects, arrays, number lines and number sentences will continue to be the main methods of recording.


Arrays also help us understand multiplying 2 and 3 digit numbers. Place value counters are used to partition the numbers into tens and ones, and the counters can make the steps quicker using our mental calculations to work out the answer.


## Pictorial:

The children will need to practise and apply these skills in a variety of real life problems. They draw pictures of the objects they have used. Often when we have a word problem we put the information into a bar to make sure we understand what we need to do. This doesn't work out the answer for us but helps us see what we need to do.


Once pupils begin to multiply one-digit by two-digit numbers this will be by using partitioning. Pupils will be unlikely to have used brackets at this stage and it is best to let them record without brackets, but with a clear understanding of what they are doing, based on an understanding of arrays and a diagram to explain the calculation.

## $8 \times 23$

| $\boldsymbol{\otimes}$ | 6 | ¢ | $\theta$ | $\theta$ | O | O | 6 | 0 | O | 9 | 0 | $\bigcirc$ | $\boldsymbol{\theta}$ | $\theta$ | 6 |  | 9 | $\theta$ | . | ¢ | $\theta$ | ¢ | ¢ | $\theta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\theta$ | $\theta$ | ¢ | ¢ | O | ¢ | - | $\checkmark$ | ${ }^{*}$ |  | $\theta$ | $\bigcirc$ | $\theta$ | ¢ | ¢ | 6 | - | $\theta$ | $\theta$ | $\theta$ | 6 | 9 | $\oplus$ | 6 | - |
| $\Theta$ | 6 | ¢ | ¢ | O | ¢ | ¢ |  | ${ }^{6}$ |  | $\theta$ | ¢ | $\bigcirc$ | $\theta$ | ¢ | - |  | $\theta$ | * | - | ¢ | * | ¢ | \% |  |
| $\boldsymbol{\theta}$ | $\theta$ | ¢ | ¢ | ¢ | ¢ | ¢ | ${ }^{6}$ | ${ }^{6}$ |  | $\theta$ | 9 | O | ¢ | ¢ | - |  | $\odot$ | $\theta$ | $\theta$ | $\bigcirc$ | ¢ | 9 | $\bigcirc$ | 2 |
| ¢ | $\theta$ | ¢ | O | * | Q | * |  | ${ }^{\circ}$ |  | $\theta$ | $\bigcirc$ | * | 6 | O | - |  | $\theta$ | $\theta$ | $\theta$ | 6 | 9 | $\stackrel{\square}{6}$ | ¢ | 9 |
| ¢ | 6 | $\theta$ | O | ¢ | ¢ | ¢ | $\checkmark$ | $\bigcirc$ |  | $\theta$ | $\bigcirc$ | O | $\theta$ | + | 6 |  | ¢ | $\Theta$ | $\theta$ | ¢ | $\theta$ | ¢ | ¢ | - |
| $\odot$ | $\theta$ | Q | Q | Q | Q | ¢ | $\checkmark$ | $\bigcirc$ |  | $\bigcirc$ | $\odot$ | $\bigcirc$ | ๑ | Q | 6 | - | ¢ | ¢ | $\theta$ | $\bigcirc$ | ¢ | $\stackrel{\rightharpoonup}{6}$ | ¢ |  |
| $\boldsymbol{\otimes}$ | 6 | © | O | ¢ | $\theta$ | 6 |  | 16 |  | $\theta$ | $\boldsymbol{\theta}$ | 0 | ¢ | $\theta$ | 6 | - | ¢ | $\Theta$ | O | ¢ | 9 | ¢ | ¢ |  |

$8 \times 23=8 \times 10+8 \times 10+8 \times 3=80+80+24=184$

## Abstract:



Drawing their number line hops here represents $3 \times 4=12$.


And here represents $4 \times 15=60$.
The earlier work on pictorial arrays leads to the grid method of multiplication:


Once children can show an understanding of a 1-digit by 2-digit multiplication both with an array and a grid multiplication they can explore multiplying a multiple of 10 by a 1-digit number. Using this decreases the number of steps needed to complete the multiplication.

| $\mathbf{X}$ | 20 | 3 |  |
| :---: | :---: | :---: | :---: |
| $\mathbf{8}$ | 160 | 24 | $=184$ |

The grid method can then be used for 2-digit by 2-digit multiplication. At first just use numbers between 11 and 19. For instance try $16 \times 14$ :

| $\mathbf{X}$ | $\mathbf{1 0}$ | $\mathbf{6}$ |  |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 0}$ | 100 | 60 |  |
| $\mathbf{4}$ | 40 | 24 |  |
|  | $=140$ | $=84$ | $=224$ |


| $\mathbf{X}$ | $\mathbf{1 0}$ | $\mathbf{6}$ |  |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 0}$ | 100 | 60 | $=160$ |
| $\mathbf{4}$ | 40 | 24 | $=64$ |
|  |  |  | $=224$ |

Later children can move on to other 2-digit numbers and decimals.

| $\mathbf{X}$ | $\mathbf{6 0}$ | $\mathbf{6}$ |  |
| :---: | ---: | ---: | ---: |
| $\mathbf{3 0}$ | 1800 | 180 |  |
| $\mathbf{4}$ | 240 | 24 |  |
|  | $=2040$ | $=204$ | $=2244$ |

$66 \times 34$
$73.5 \times 17$

| $\mathbf{X}$ | $\mathbf{7 0}$ | $\mathbf{3}$ | $\mathbf{0 . 5}$ |  |
| :---: | ---: | :---: | :---: | :---: |
| $\mathbf{1 0}$ | 700 | 30 | 5 |  |
| $\mathbf{7}$ | 490 | 21 | 3.5 |  |
|  | $=1190$ | $=51$ | $=8.5$ |  |

Moving children to a vertical multiplication calculation needs to be done with care, ensuring that they understand what they are doing and why they are doing it. To start with it's important that all the steps that would occur in the grid method are replicated in the vertical one.

| $\mathbf{X}$ | $\mathbf{1 0 0}$ | $\mathbf{7 0}$ | $\mathbf{3}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2 0}$ | 2000 | 1400 | 60 | $=3460$ |
| $\mathbf{7}$ | 700 | 490 | 21 | $=1211$ |
|  |  |  |  | $=4671$ |



For many children it's best to stay at this level, whilst developing their ability to calculate with decimals and in a range of contexts. Moving on to more compact methods requires excellent estimation, place value and mental calculation skills and should not be attempted unless children have these skills and can learn the 'shortcut' method in one or two lessons. Showing children the compact form will suit some but do not teach unless they quickly show understanding as will lead to confusion.


## Division

Key language: share, group, divide, divided by, half, array, divisor, factor, divisible by, number sentence.

## Concrete:

The children usually have a well-developed sense of sharing fairly, and will use real life objects to share into groups, or into sorting boxes or onto sharing circles.


We also introduce division as repeated addition using Numicon and number lines. Children need to see division as sharing and grouping - how many groups of 3 can I make from 12?


As with multiplication, division is recorded with objects, arrays, number lines or number sentences.


I start at zero and count in 8 s until I get to 16. That's two eights.
Calculations with remainders in the quotient are also recorded on a number line.
$17 \div 8=2$ with 1 left over


I start at zero and count in 8 s until I get to 16 . Then there is 1 more to get to 17 , so I have 2 jumps of 8 and 1 left over (remainder). It's important that the remainder is never recorded as a jump as the jumps show how many eights have been made. Using a cross for each number left over tends to work well.

Using arrays helps with division too. Using our tables is important. We start with numbers which leave no remainders first. Sometimes we draw dots for arrays.


| 10 s | 1s |
| :---: | :---: |
|  |  |
|  |  |
|  |  |


| 10s | 1s |
| :---: | :---: |
| $\odot$ | 0000 |
| $\odot$ | 0000 |
| $\odot$ | $000 \bullet$ |


|  | 10s | 1s |
| :---: | :---: | :---: |
|  | $\bigcirc$ |  |
| $\longrightarrow$ | $\bigcirc$ |  |
|  | $\bigcirc$ |  |
|  | $\bigcirc 00$ |  |
|  | 10s | 1s |
| =14 | - |  |
|  | $\bigcirc$ |  |
|  | $\bigcirc$ |  |

Children use place value counters to solve $42 \div 3=14$

## Pictorial:

The children will again need to practice and apply these skills in a variety of real life contexts and using different measures. They draw pictures of the objects they have used, such as counters, place value counters or base 10. Often when we have a word problem we put the information into a bar to make sure we understand what we need to do. This doesn't work out the answer for us but helps us see what we need to do.

$6 \div 2=3$
3


$615 \div 3=123$

## Abstract:

Children can show their calculations using abstract representations such as empty number lines. When children are dividing numbers which are more than 10 times the divisor it becomes useful to work with multiples of the divisor. In this example children would count in steps of 70 , showing 7 ten times equals 70 , then deciding how to do the next step of $56 \div 7$. It could be one jump of 7 eight times, or could be smaller jumps of $7,14,21$ and so on until the 196 is reached.
$196 \div 7=$


For many pupils, the addition of an 'I Know' box makes the calculation easy.
For $259 \div 6$ there are a couple of ways of doing the 'I know' box.

## I Know

$6 \times 10=60$
$6 \times 20=120$
$6 \times 30=180$
$6 \times 40=240$
$6 \times 50=300$ too many
so I will use $6 \times 40$

I Know
$6 \times 2=12$
$6 \times 5=30$
$6 \times 10=60$
$6 \times 20=120$
$6 \times 50=300$ too many

These are easy mental calculations and children are less likely to make calculation errors. The final calculation will vary depending on which facts the child uses.


Key understanding - Children must always record what happens in their own mind, and not try to guess and record what's in yours.

When dividing decimals it is useful to begin by adapting a calculation that can already be understood.

| I Know |  |  |
| :--- | :--- | :--- |
| $6 \times 10=60$ | $6 \times 1=6$ | $6 \times 0.1=0.6$ |
| $6 \times 20=120$ | $6 \times 2=12$ | $6 \times 0.2=1.2$ |
| $6 \times 50=300$ | $6 \times 5=30$ | $6 \times 0.5=3$ |
| $6 \times 60=360$ | $6 \times 6=36$ | $6 \times 0.6=3.6$ |


$348 \div 6=58$

$34.8 \div 6=5.8$

## Long Division-

$1736 \div 14$

I know
$2 \times 14=284 \times 14=56$
$5 \times 14=70$
$10 \times 14=140$
$20 \times 14=280$


## Compact Methods- Representations.



## Further Guidance.

Videos showing these strategies in use, with different year groups are available on the NCETM website:

## https://www.ncetm.org.uk/in-the-classroom/support-for-schools-addressing-ongoing-coronavirus-impact/primary-video-lessons/

Progression maps showing the coverage in each year group are also available to aid planning. Alongside this are examples of developing reasoning activities to help improve depth and understanding. These also will be helpful in formative assessments.

A glossary of terms used in the programmes of study can be accessed on this website to ensure continuity and upskill teacher's subject knowledge and to support parents.

## Home School Partnership

Parents are supported by having access to the Maths page on the website. There are four Guides for Parents on the Maths page of the Curriculum tab of the website which show, via a comprehensive series of photos, the common representations used in each phase of the key stages. These will make clear which methods are suggested for each year group. However it is important that the child uses the representations that are meaningful to them, even if these are from a different year group.

Children can also access a maths bag of resources via a library loan for the year. Each bag has practical equipment that are used in school with that year group and there is a Parent Guide in the bag to show how to use the equipment. KS2 bags have a book in the bag which can be used as reference on the calculation methods and expectations for learning in that year group.

Children also have a login which can be used at home for ttrockstars from Year 1 to 6, to help them to learn their times tables. Children may also be given tables challenges as part of their homework to support them to learn their times tables by the end of Year 4.

